Int. J. Multiphase Flow. Vol. 4. pp. 89–95. Pergamon/Elsevier. 1978. Printed in Great Britain.

THE EROSION OF DUST BY A SHOCK WAVE IN AIR: INITIAL STAGES WITH LAMINAR FLOW

W. MERZKIRCH and K. BRACHT
Institut für Thermo- und Fluiddynamik, Ruhr-Universität, D-4630 Bochum, West Germany

(*Received* 20 *March* 1977)

Abstract—The erosion of dust by a shock wave in air and by the subsequent air flow was investigated theoretically and experimentally. The paths of single particles were calculated for the initial state of erosion when the flow in the shock tube boundary layer was still laminar. High-speed cinematographic experiments performed with a shock tube yielded mapping of the development of the dust cloud. From the agreement between the measured height of the cloud and the calculated height of flight of the particles one can conclude that the assumed model for the motion of the particles adequately describes the removal of particles from the wall.

1. INTRODUCTION

The erosion of dust by an air shock wave and the dispersion of dust in the subsequent air flow have been studied in association with safety requirements in coal mines. The problem is to analyze the formation of clouds of dispersed coal dust which, upon ignition, may explode and cause severe hazards in mines. Laboratory experiments performed with shock tubes (Gerrard 1963; Fletcher 1976) supplemented the data from large scale tests, and helped in gaining more insight into the fundamental mechanisms of the problem. In such experiments the dust is initially deposited on the floor of the tube and the development of the dust cloud after passage of a shock wave is measured.

For the idealized conditions of the respective shock tube experiment, the shock wave is a plane front moving in a direction perpendicular to its plane and parallel to the tube axis. The shock wave initiates an air flow parallel to the tube axis and with only one velocity component, $u_G(x, y, z)$. The tube axis is taken to be parallel to the x-direction. With gas dynamic relationships one determines a value $u_G = u_\infty = \text{const}$ for the (undisturbed) gas velocity behind the shock wave, with u_∞ depending on the shock velocity u_s, and only in the wall boundary layer (called the shock tube boundary layer) u_G is not a constant, $u_G = u_G(x, y, z)$. A fundamental problem for the described experiments is to explain and predict the forces which cause the dust particles to rise from the tube bottom into the air flow which is parallel to the initial dust layer. Gerrard (1963) explained these forces as a result of shock wave diffraction and reflection in the dust layer. Examination of a shadowgraph of the interaction of a moving air shock with a plane dust layer shows indeed a curvature of the initially plane shock front close to the bottom (figure 1). This indicates shock wave diffraction and the penetration of pressure energy into the dust layer. However, from quantitative experiments performed by Fletcher (1976) one must conclude that this diffraction effect cannot adequately describe the lift forces acting on the dust particles which later form the dust cloud.

In the present paper an attempt is made to relate the lift of the dust particles to an interaction between these particles and the shear flow in the shock tube boundary layer. In the regime adjacent to the moving shock wave the boundary layer flow is laminar. At the same time, the boundary layer is very thin, thus producing strong velocity gradients. Dust particles in this shear flow regime experience a lift force in the direction of the air velocity gradient. An analytical solution by Saffman (1965) for this lift force is used in the equations of motion for single dust particles, and the equations are treated to yield the paths of such particles. It is assumed that this lift force and the aerodynamic drag are the only forces acting on the first particles which are raised from the initially plane dust layer. These particles then form the upper edge of the developing dust cloud. The rising dust is photographed in a series of shock tube experiments, and the measured edge of the dust cloud is compared with the calculated

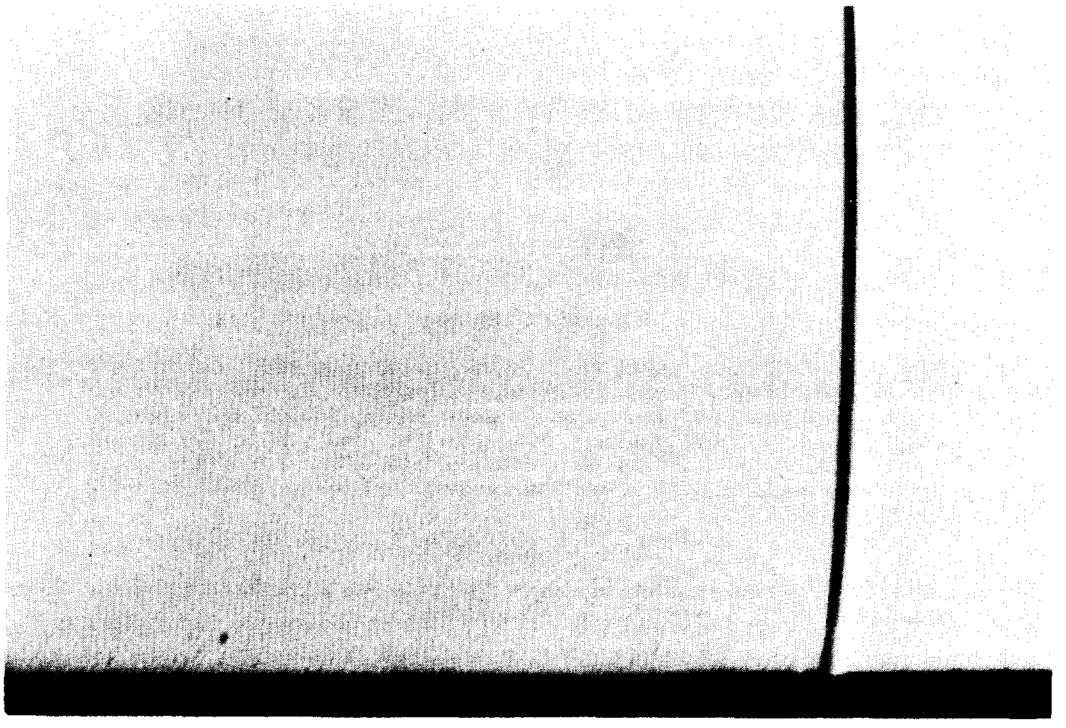

Figure 1. Shadowgraph of a shock wave moving from the left to the right over a plane dust layer. The lower part of the shock front appears curved due to diffraction at the dust particles.

particle paths. This comparision can yield a check of the assumed model for the particle lift and motion.

2. CALCULATION OF THE MOTION OF SINGLE PARTICLES

The paths of single particles lying initially on the floor of the channel are calculated by solving the respective equations of motion in the x, y-plane, i.e. the problem is treated as being two-dimensional. The particles are assumed to be spherical with diameter D_p, their material having the density ρ_p. The shock wave moves into the x-direction with a velocity u_s, and the gas (air) behind the shock flows with an undisturbed velocity $u_G = u_\infty$, the value of u_∞ solely depending on u_s. In the shock tube boundary layer, the gas velocity at a station x depends on the y-coordinate and on the time coordinate t or on the shock wave distance to that point x.

All physical quantities are brought to a non-dimensional form (designated by an asterisk) by relating all lengths to the particle diameter D_p, all velocities to the velocity difference $u_D = u_s - u_\infty$, and by introducing the ratio of the densities of the gas (air) and that of the particle material: $\rho^* = \rho_G/\rho_p$. The definitions of the non-dimensional forms for the constant of gravity g and the kinematic viscosity ν are:

$$g^* = g \cdot D_p/u_D^2 \qquad \text{and}$$

$$\nu^* = \nu/D_p \cdot u_D.$$

The time coordinate t is brought to a non-dimensional form through

$$t^* = u_D \cdot t/D_p.$$

In the x-component of the equation of motion one takes account of the aerodynamic drag as the only force acting on the particle in this direction. Writing for the mass of the particle $m_p = (\pi/6)\rho_p D_p^3$, and designating the drag coefficient by c_D, one arrives at the following non-dimensional form of this equation:

$$\frac{du_p^*}{dt^*} = \tfrac{3}{4}\, c_D \rho^* (u_G^* - u_p^*) \cdot \sqrt{(u_G^* - u_p^*)^2 + v_p^{*2}}. \qquad [1]$$

The velocity components of the particle, u_p^* and v_p^* are functions of x^*, y^*, t^*.

The acceleration of the particle in the y-direction is due to the action of gravity, the respective component of the aerodynamic drag, and the aforementioned lift force in a shear flow as analyzed by Saffman (1965). The non-dimensional form of this component of the equation of motion is

$$\frac{dv_p^*}{dt^*} = -g^* - \frac{3}{4} c_D \rho^* v_p^* \sqrt{[(u_G^* - u_p^*)^2 + v_p^{*2}]} + \frac{3\bar{a}}{2\pi} \rho^* (u_G^* - u_p^*) \sqrt{\left(v^* \left|\frac{\partial u_G^*}{\partial y^*}\right|\right)}. \qquad [2]$$

The value of the constant $\bar{a}$ is taken from Halow & Wills (1970) as 32.2. The drag coefficient for a sphere c_D is a function of the particle Reynolds number and is defined as

$$c_D = f(Re_p) = f\left(\frac{D_p \sqrt{(u_G - u_p)^2 + v_p^2}}{\nu}\right). \qquad [3]$$

[1] and [2] are two equations to determine the two unknowns u_p^*, v_p^*. The gas velocity u_G in the laminar boundary layer flow can be described, according to Schlichting (1968) by:

$$u_G^* = u_\infty^* \sin\left(\frac{\pi}{2}\frac{y^*}{\delta^*}\right) \qquad [4]$$

where δ^* is the (non-dimensional) instantaneous boundary layer thickness. The value of δ in a point x depends on the distance between x and the instantaneous position of the shock wave. δ^* therefore is a function of x^*, t^* and the shock velocity u_s^*. For a laminar boundary layer on a flat plate one derives

$$\delta^* = 3.64 \cdot \sqrt{v^*(u_s^* t^* - x^*)}. \qquad [5]$$

This makes clear that the time coordinate t^* is defined as $t^* = 0$ for the instant when the moving shock wave reaches the original position $x^* = 0$ of the considered particle.

In order to solve [1] and [2] for u_p^*, v_p^*, one has to know the function $f(Re_p)$ in [3]. The familar relationship between c_D for a sphere and Re_p is an empirical result and not available in explicit form. For the range of Reynolds numbers in the present problem one may use a representation of c_D as described by Morsi & Alexander (1972).

Equations [1] and [2] are now integrated with a numerical Runge–Kutta-procedure. For this purpose it is required to define the initial condition for the motion of the considered particle. As stated above, $t^* = 0$ is the time of arrival of the shock wave at the original position of the particle, $x^* = 0$. In $x^* = 0$, the gas begins to flow for $t^* \geqslant 0$; but it is assumed that the particle starts moving only at a later time $t_0^* > 0$ when the boundary-layer thickness has reached the size of the particle diameter, i.e. $t^* = t_0^*$ for $\delta^* = 1$. The motion of the particle is calculated stepwise, the particle Reynolds number is determined for each step in order to match the respective value of the drag coefficient c_D.

Equations [1]–[5] involve four arbitrary parameters: ρ^*, v^*, u_s^* and g^*. Calculations have shown that, for the short period of time considered in this study, the influence of the force of gravity on the particle motion is negligibly small. Therefore, one may cancel the term $-g^*$ in [2] which reduces the number of arbitrary parameters to three. The following physical significance can be attributed to these parameters: ρ^* describes the material of the particle, v^*

characterizes the gas, and u_s^* is a measure of the shock wave intensity or of the velocity of the undisturbed gas flow. This study is restricted to the first particles which are raised in the erosion process due to the aerodynamic lift force generated in the laminar boundary layer. This restriction justifies the assumption that—due to the low particle concentration—interaction forces between particles can be neglected, and that the flow of the gas is not noticeably disturbed by the presence of the particles.

3. SHOCK TUBE EXPERIMENTS

Experiments have been performed in an air shock tube in order to photograph the development of the dust cloud. The shock tube has an optical test chamber with viewing windows. The tube is of rectangular cross-section of 40×60 mm. A cavity in the floor of the test section, 300 mm long, is filled with dust (figure 2). Before every experiment the dust surface is spread to be smooth and plane with the rest of the tube floor. The erosion of dust in the air flow behind the shock wave is photographed in form of single spark exposures. Repeating the experiments and triggering the spark with a certain delay with respect to the earlier exposure delivers a cinematographic recording of the formation of the dust cloud. For several reasons it was not possible to use coal dust in these laboratory experiments. Instead, a dust was used which is normally utilized in fire extinguishers and which has a size distribution similar to that of coal dust. The density of this dust is $\rho_p = 2.9\,\text{g/cm}^3$. The experiments were performed with relatively small shock strengths, the first resulting in a shock velocity $u_s = 434$ m/sec and a gas velocity $u_\infty = 133$ m/sec, the second for $u_S = 385$ m/sec and $u_\infty = 64$ m/sec. The photographs obtained from these experiments can be used to measure the height of the dust cloud as a function of time.

4. RESULTS AND DISCUSSION

The numerical integration of [1] and [2] yields the velocity components u_p^*, v_p^* as a function of the dimensionless time coordinate t^*. For two fixed parameters, namely $\rho^* = 6.2 \cdot 10^{-4}$ and $u_s^* = 1.44$, figure 3 shows how the velocity of the particles in x-direction approaches the velocity of the gas with increasing time. For a specific gas and constant shock strength, the parameter ν^* depends solely on the particle diameter D_p. As one would expect, the approach to the gas velocity, i.e. $u_p^*/(u_s^* - 1) \rightarrow 1$, is the faster the smaller the particle.

The main interest in the present investigations is in the particle motion in y-direction. Calculations and experiments are performed to check the validity of the assumed model for the aerodynamic lift force acting on the particles in the shock tube boundary layer. The v_p^*-profiles in figure 4, calculated with the same parametric values as in figure 3, start at various time coordinates $t_0^* > 0$ according to the size of the particle and as explained in section 2. The acceleration in y-direction takes place only within the boundary layer. The smaller particles experience higher values for both acceleration and maximum velocity than the larger ones. Outside of the boundary layer the particles are decelerated due to the action of the aerodynamic drag, while the influence of gravity is not yet important in these early stages, i.e. $g^* \approx 0$, as mentioned earlier. Inertia effects cause the degree of decelaration to be the lower, the larger

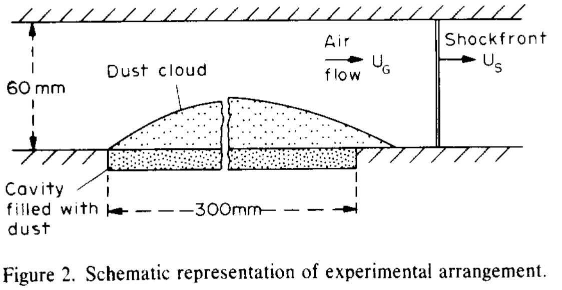

Figure 2. Schematic representation of experimental arrangement.

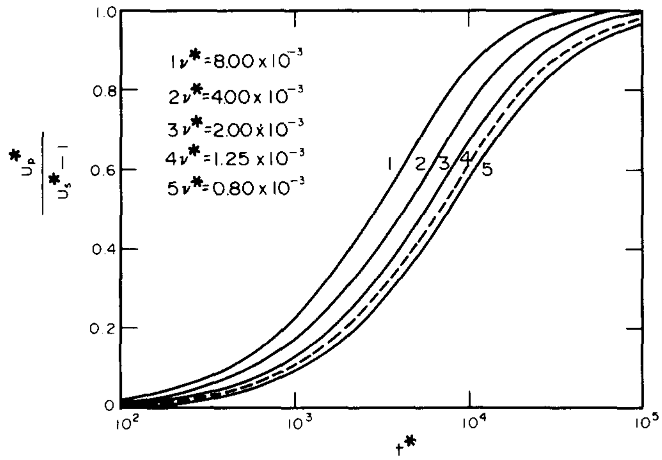

Figure 3. Approach of the horizontal component of particle velocity to the undisturbed gas velocity as a function of time. The dashed curve with parameter $v^* = 1.25 \cdot 10^{-3}$ refers to a particle with $D_p = 30\mu$ and the experimental conditions of figure 5.

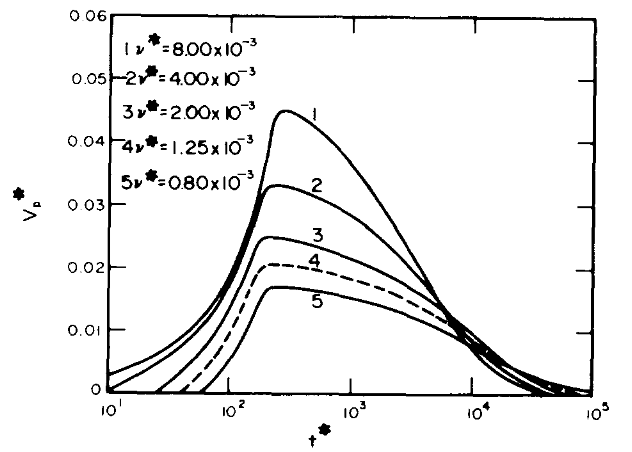

Figure 4. Vertical component of particle velocity as a function of time. The dashed curve with parameter $v^* = 1.25 \cdot 10^{-3}$ refers to a particle with $D_p = 30\mu$ and the experimental conditions of figure 5.

the particle. Large particles therefore continue to move with positive values of v_p for a longer duration than small particles.

For the purpose of better interpretation and comparison with measured results, the heights of flight of the dust particles are presented in figure 5 in dimensional form. The abscissa designates the distance between the position of a particle during flight and the instantaneous position of the shock wave. Particle paths are shown for four different particle diameters. The fixed values for shock velocity, undisturbed gas velocity, densities of gas and dust, etc. correspond with the experimental conditions in one of the mentioned test series. The dependence of the initial acceleration on particle size is again evident. Due to its rapid deceleration outside the boundary layer, the 10 μ-particle reaches only a moderate value for the maximum height of flight. This maximum value increases with increasing particle diameter. But for this tendency there is apparently a limiting value of D_p, since the curve for $D_p = 50\ \mu$ lies under that for $D_p = 30\ \mu$. The limiting value in this example is near $D_p = 30\ \mu$; for particles larger than this limiting value the maximum height of flight will decrease with increasing particle diameter. This change in tendency can well be understood since a very large or heavy particle might not be lifted at all.

Also shown in figure 5 are experimental results which indicate the measured heights of the dust cloud obtained under these specific experimental conditions (see section 3). The dust used in the experiments has a size distribution $D_p \leq 40\ \mu$. If one assumes that the upper edge of the dust cloud is formed by those particles which gain the maximum height of flight, then figure 5 may be interpreted to prove good agreement between experiment and theory.

The same conclusion may be drawn from the results of figure 6. Calculated profiles for height of flight and measured heights of the dust cloud refer to a weaker shock wave than in the foregoing example, and therefore to a lower velocity of the undisturbed flow outside of the boundary layer.

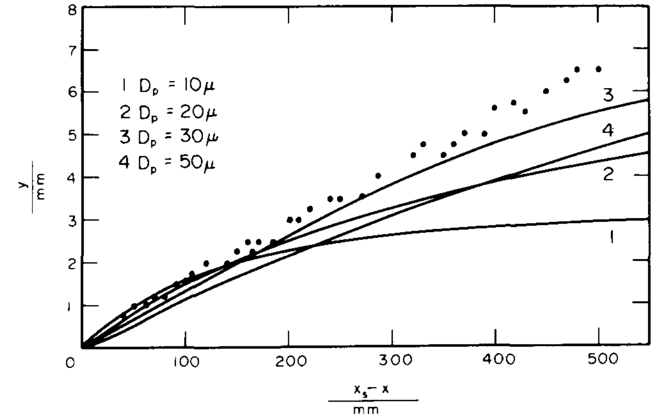

Figure 5. Heights of flight, y, for different sized particles of a dust with $\rho_p = 2.9\ g/cm^3$. The scale of the abscissa is the distance to the instantaneous position of the shock front. Undisturbed air velocity $u_\infty = 133$ m/sec. The points indicate measured heights of the dust cloud as photographed in the shock tube experiments.

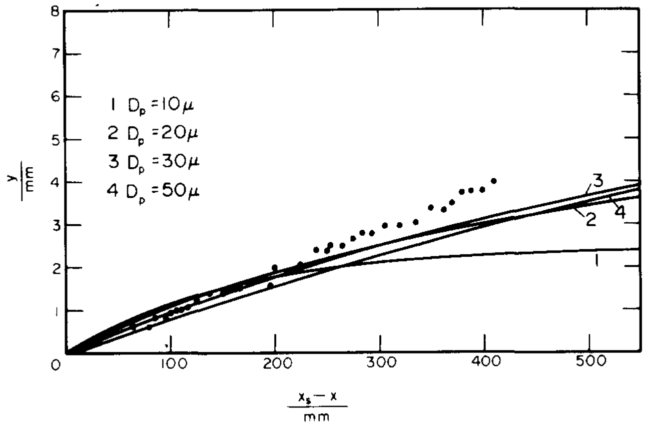

Figure 6. Same as figure 5 with $u_\infty = 64$ m/sec.

As a result of the present investigation one may conclude that the assumed model for the motion of the particles, based on Saffman's theory for the lift of a spherical particle in a laminar shear flow, may correctly describe the initial development of the dust cloud. The comparison between experiments and calculations, as seen in figures 5 and 6, shows good agreement, particularly if one takes account of the inevitable errors in the measurements. The assumed interaction between the boundary layer flow and the spherical dust particle might not be the sole lift force which causes the particles to move from the bottom into the gas flow. But it is certainly the dominant force in this early stage of development, and other possible contributions like pressure forces due to shock reflections in the dust layer (Gerrard 1963) or Magnus forces are negligible. The magnitude of Saffman's lift force decreases rapidly with increasing thickness of the shock tube boundary layer. At the same time, the boundary layer flow has a tendency to become turbulent. Therefore, the validity of the used model for the dust motion holds only for a very short period of time within which the flow is laminar and the boundary layer thin. The length of this period cannot be derived from the present investigations. From preliminary results of an experimental program which is presently under study one may conclude that this period of validity is of the order of 2 msec. The further removal of dust from the wall must be attributed to an interaction between the (then) turbulent boundary layer and the dust layer.

Acknowledgement—This research has been supported by a grant from Minister für Wissenschaft und Forschung des Landes Nordrhein-Westfalen.

REFERENCES

FLETCHER, B. 1976 The interaction of shock with a dust deposit. *J. Phys. D: Appl. Phys.* **9**, 197–202.

GERRARD, J. H. 1963 An experimental investigation of the initial stages of the dispersion of dust by shock waves, *Br. J. Appl. Phys.* **14**, 186–192.

HALOW, J. S. & WILLS, G. B. 1970 Radial migration of spherical particles in Couette systems. *A.I.Ch.E. Jl.* **16**, 281–286.

MORSI, S. A. & ALEXANDER, A. J. 1972 An investigation of particle trajectories in two-phase flow systems. *J. Fluid Mech.* **55**, 193–208.

SAFFMAN, P. G. 1965 The lift on a small sphere in a slow shear flow. *J. Fluid Mech.* **22**, 385–400.

SCHLICHTING, H. 1968 *Boundary Layer-Theory*. McGraw-Hill, New York.